Off–Diagonal 5D Metrics and Mass Hierarchies with Anisotropies and Running Constants

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(October 16, 2001)

The gravitational equations of the three dimensional (3D) brane world are investigated for both off–diagonal and warped 5D metrics which can be diagonalized with respect to some anholonomic frames when the gravitational and matter fields dynamics are described by mixed sets of holonomic and anholonomic variables. We construct two new classes of exact solutions of Kaluza–Klein gravity which generalize the Randall–Sundrum metrics to configurations with running on the 5th coordinate gravitational constant and anisotropic dependencies of effective 4D constants on time and/or space variables. We conclude that by introducing gauge fields as off–diagonal components of 5D metrics, or by considering anholonomic frames modelling some anisotropies in extra dimension spacetime, we induce anisotropic tensions (gravitational polarizations) and running of constants on the branes. This way we can generate the TeV scale as a hierarchically suppressed anisotropic mass scale and the Newtonian and general relativistic gravity are reproduced with adequate precisions but with corrections which depend anisotropically on some coordinates.

PACS numbers: Pacs 04.90.+e, 04.50.+h,

Recent approaches to String/M–theory and particle physics are based on the idea that our universe is realized as a three brane, modelling a four dimensional, 4D, pseudo–Riemannian spacetime, embedded in the 5D anti–de Sitter (AdS_5) bulk spacetime. In such models the extra dimension need not be small (they could be even infinite) if a nontrivial warped geometric configuration, being essential for solving the mass hierarchy problem and localization of gravity, can "bound" the matter fields on a 3D subspace on which we live at low energies, the gravity propagating, in general, in a higher dimension spacetime (see Refs.: [1] for string gravity papers; [2] for extra dimension particle fields and gravity phenomenology with effective Plank scale; [3] for the simplest and comprehensive models proposed by Randall and Sundrum; here we also point the early works [4] in this line and cite [5] as some further developments with supresymmetry, black hole solutions and cosmological scenarios).

In higher dimensional gravity much attention has been paid to off–diagonal metrics beginning the Salam, Strathee and Perracci work [6] which showed that including off–diagonal components in higher dimensional metrics is equivalent to including $U(1), SU(2)$ and $SU(3)$ gauge fields. Recently, the off–diagonal metrics were considered in a new fashion by applying the method of anholonomic frames with associated nonlinear connections [7] which allowed us to construct new classes of solutions of Einstein's equations in three (3D), four (4D) and five (5D) dimensions, with generic local anisotropy (*e.g.* static black hole and cosmological solutions with ellipsoidal or torus symmetry, soliton–dilaton 2D and 3D configurations in 4D gravity, and wormhole and flux tubes with anisotropic polarizations and/or running on the 5th coordinate constants with different extensions to backgrounds of rotation ellipsoids, elliptic cylinders, bipolar and torus symmetry and anisotropy.

The point of this paper is to argue that if the 5D gravitational interactions are parametrized by off–diagonal metrics with a warped factor, which could be related with an anholonomic higher dimensional gravitational dynamics and/or with the fact that the gauge fields are included into a Salam–Strathee –Peracci manner, the fundamental Plank scale M_{4+d} in $4+d$ dimensions can be not only considerably smaller than the the effective Plank scale, as in the usual locally isotropic Randall–Sundrum (in brief, RS) scenarios, but the effective Plank constant is also anisotropically polarized which could have profound consequences for elaboration of gravitational experiments and for models of the very early universes.

We will give two examples with one additional dimension ($d=1$) when an extra dimension gravitational anisotropic polarization on a space coordinate is emphasized or, in the second case, a running of constants in time is modelled. We will show that effective gravitational Plank scale is determined by the higher-dimensional curvature and anholonomy of pentad (funfbein, of frame basis) fields rather than the size of the extra dimension. Such curvatures and anholonomies

are not in conflict with the local four-dimensional Poincare invariance.

We will present a higher dimensional scenario which provides a new RS like approach generating anisotropic mass hierarchies. We consider that the 5D metric is both not factorizable and off–diagonal when the four-dimensional metric is multiplied by a "warp" factor which is a rapidly changing function of an additional dimension and depend anisotropically on a space direction and runs in the 5-th coordinate.

Let us consider a 5D pseudo–Riemannian spacetime provided with local coordinates $u^\alpha = (x^i, y^a) = (x^1 = x, x^2 = f, x^3 = y, y^4 = s, y^5 = p)$, where $(s,p) = (z,t)$ (Case I) or, inversely, $(s,p) = (t,z)$ (Case II) – or more compactly $u = (x,y)$ – where the Greek indices are conventionally split into two subsets x^i and y^a labeled respectively by Latin indices of type $i,j,k,... = 1,2,3$ and $a,b,... = 4,5$. The local coordinate bases, $\partial_\alpha = (\partial_i, \partial_a)$, and their duals, $d^\alpha = \left(d^i, d^a\right)$, are defined respectively as

$$\partial_\alpha \equiv \frac{\partial}{du^\alpha} = (\partial_i = \frac{\partial}{dx^i}, \partial_a = \frac{\partial}{dy^a}) \text{ and } d^\alpha \equiv du^\alpha = (d^i = dx^i, d^a = dy^a). \tag{1}$$

For the 5D (pseudo) Riemannian interval $dl^2 = G_{\alpha\beta} du^\alpha du^\beta$ we choose the metric coefficients $G_{\alpha\beta}$ (with respect to the coordinate frame (1)) to be parametrized by a off–diagonal matrix (ansatz)

$$\begin{bmatrix} g + {w_1}^2 h_4 + {n_1}^2 h_5 & w_1 w_2 h_4 + n_1 n_2 h_5 & w_1 w_3 h_4 + n_1 n_3 h_5 & w_1 h_4 & n_1 h_5 \\ w_1 w_2 h_4 + n_1 n_2 h_5 & 1 + {w_2}^2 h_4 + {n_2}^2 h_5 & w_2 w_3 h_4 + n_2 n_3 h_5 & w_2 h_4 & n_2 h_5 \\ w_1 w_3 h_4 + n_1 n_3 h_5 & w_3 w_2 h_4 + n_2 n_3 h_5 & g + {w_3}^2 h_4 + {n_3}^2 h_5 & w_3 h_4 & n_3 h_5 \\ w_1 h_4 & w_2 h_4 & w_3 h_4 & h_4 & 0 \\ n_1 h_5 & n_2 h_5 & n_3 h_5 & 0 & h_5 \end{bmatrix} \tag{2}$$

where the coefficients are some necessary smoothly class functions of type:

$$g = g(f,y) = a(f)\ b(y), h_4 = h_4(f,y,s) = \eta_4(f,y) g(f,y) q_4(s),$$
$$h_5 = h_5(f,y,s) = g(f,y) q_5(s), w_i = w_i(f,y,s), n_i = n_i(f,y,s).$$

The metric (2) can be equivalently rewritten in the form

$$\delta l^2 = g_{ij}\left(f,y\right) dx^i dx^i + h_{ab}\left(f,y,s\right) \delta y^a \delta y^b, \tag{3}$$

with diagonal coefficients

$$g_{ij} = \begin{bmatrix} g & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & g \end{bmatrix} \text{ and } h_{ab} = \begin{bmatrix} h_4 & 0 \\ 0 & h_5 \end{bmatrix} \tag{4}$$

if instead the coordinate bases (1) one introduce the anholonomic frames (anisotropic bases)

$$\delta_\alpha \equiv \frac{\delta}{du^\alpha} = (\delta_i = \partial_i - N_i^b(u)\ \partial_b, \partial_a = \frac{\partial}{dy^a}) \text{ and } \delta^\alpha \equiv \delta u^\alpha = (\delta^i = dx^i, \delta^a = dy^a + N_k^a(u)\ dx^k) \tag{5}$$

where the N–coefficients are parametrized $N_i^4 = w_i$ and $N_i^5 = n_i$.

In this paper we consider a slice of AdS_5 provided with an anholnomic frame structure (5) satisfying the relations $\delta_\alpha \delta_\beta - \delta_\beta \delta_\alpha = W_{\alpha\beta}^\gamma \delta_\gamma$, with nontrivial anholonomy coefficients

$$W_{ij}^k = 0, W_{aj}^k = 0, W_{ia}^k = 0, W_{ab}^k =, W_{ab}^c = 0, W_{ij}^a = \delta_i N_j^a - \delta_j N_i^a, W_{bj}^a = -\partial_b N_j^a, W_{ia}^b = \partial_a N_j^b.$$

We assume there exists a solution of 5D Einstein equations with 3D brane configuration that effectively respects the local 4D Poincare invariance with respect to anholonomic frames (5) and that the metric ansatz (2) (equivalently, (4)) transforms into the usual RS solutions

$$ds^2 = e^{-2k|f|} \eta_{\underline{\mu}\underline{\nu}} dx^{\underline{\mu}} dx^{\underline{\nu}} + df^2 \tag{6}$$

for the data: $a(f) = e^{-2k|f|},\ k = const,\ b(y) = 1, \eta_4(f,y) = 1, q_4(s) = q_5(s) = 1, w_i = 0, n_i = 0$, where $\eta_{\underline{\mu}\underline{\nu}}$ and $x^{\underline{\mu}}$ are correspondingly the diagonal metric and Cartezian coordinates in 4D Minkowski spacetime and the extra-dimensional coordinate f is to be identified $f = r_c \phi$, ($r_c = const$ is the compactification radius, $0 \leq f \leq \pi r_c$) like in the first work [3] (or 'f" is just the coordinate 'y' in the second work [3]).

The set-up for our model is a single 3D brane with positive tension, subjected to some anholonomic constraints, embedded in a 5D bulk spacetime provided with a off–diagonal metric (2). In order to carefully quantize the system, and treat the non-normalizable modes which will appear in the Kaluza-Klein reduction, it is useful to work with respect to anholonomic frames were the metric is diagonalized by corresponding anholonomic transforms and is necessary to work in a finite volume by introducing another brane at a distance πr_c from the brane of interest, and taking the branes to be the boundaries of a finite 5th dimension. We can remove the second brane from the physical set-up by taking the second brane to infinity.

The action for our anholonomic funfbein (pentadic) system is

$$S = S_{gravity} + S_{brane} + S_{brane'} \tag{7}$$
$$S_{gravity} = \int \delta^4 x \int \delta f \sqrt{-G}\{-\Lambda(f) + 2M^3 R\},\ S_{brane} = \int \delta^4 x \sqrt{-g_{brane}}\{V_{brane} + \mathcal{L}_{brane}\},$$

where R is the 5D Ricci scalar made out of the 5D metric, $G_{\alpha\beta}$, and Λ and V_{brane} are cosmological terms in the bulk and boundary respectively. We write down $\delta^4 x$ and δf, instead of usual differentials d^4x and df, in order to emphasize that the variational calculus should be performed by using N–elongated partial derivatives and differentials (5). The coupling to the branes and their fields and the related orbifold boundary conditions for vanishing N–coefficients are described in Refs. [3] and [8].

The Einstein equations, $R^{\alpha}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta} R = \Upsilon^{\alpha}_{\beta}$, for a diagonal energy–momentum tensor $\Upsilon^{\beta}_{\alpha} = [\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5]$ and following from the action (7) and for the ansatz (2) (equivalently, (4)) with $g = a(f)b(y)$ transform into

$$\frac{1}{a}\left[a_1'' - \frac{(a')^2}{2a}\right] + \frac{\beta}{h_4 h_5} = 2\Upsilon_1,\ \frac{(a')^2}{2a} + \frac{P(y)}{a} + \frac{\beta}{h_4 h_5} = 2\Upsilon_2(f), \tag{8}$$
$$\frac{1}{a}\left[a'' - \frac{(a')^2}{2a}\right] + \frac{P(y)}{a} = 2\Upsilon_4,\ w_i\beta + \alpha_i = 0,\ n_i^{**} + \gamma n_i^{*} = 0,$$

where

$$\alpha_1 = \ h_5^{*\bullet} - \frac{h_5^*}{2}\left(\frac{h_4^\bullet}{h_4} + \frac{h_5^\bullet}{h_5}\right)\ \alpha_2 = h_5^{*'} - \frac{h_5^*}{2}\left(\frac{h_4^{'}}{h_4} + \frac{h_5^{'}}{h_5}\right), \alpha_3 = h_5^{*\#} - \frac{h_5^*}{2}\left(\frac{h_4^{\#}}{h_4} + \frac{h_5^{\#}}{h_5}\right),$$
$$\beta = h_5^{**} - \frac{(h_5^*)^2}{2h_5} - \frac{h_5^* h_4^*}{2h_4}, P = \frac{1}{b^2}\left[b^{\#\#} - \frac{(b^{\#})^2}{b}\right],\ \gamma = \frac{3}{2}\frac{h_5{}^*}{h_5} - \frac{h_4{}^*}{h_4}, \tag{9}$$

the partial derivatives are denoted: $h^\bullet = \partial h/\partial x^1, h' = \partial h/\partial x^2, h^{\#} = \partial h/\partial x^3,\ h^* = \partial h/\partial s.$

Our aim is to construct a metric

$$\delta s^2 = g\,(f,y)\,[dx^2 + dy^2 + \eta_4\,(f,y)\,\delta s^2 + q_5(s)\delta p^2] + df^2, \tag{10}$$

with the anholonomic frame components defined by 'elongation' of differentials, $\delta s = ds + w_2 df + w_3 dy$, $\delta p = dp + n_1 dx + n_2 df + n_3 dy$, and the "warp" factor being written in a form similar to the RS solution

$$g\,(f,y) = a(f)b(y) = \exp[-2k_f|f| - 2k_y|y|], \tag{11}$$

which defines anisotropic RS like solutions of 5D Einstein equations with variation on the 5th coordinate cosmological constant in the bulk and possible variations of induced on the brane cosmological constants.

By straightforward calculations we can verify that a class of exact solutions of the system of equations (8) for $P(y) = 0$ (see (9)) :

$$h_4 = g(f,y), h_5 = g(f,y)\rho^2(f,y,s),$$

were

$$\begin{aligned}\rho(f,y,s) &= |\cos\tau_+\,(f,y)\,|,\ \tau_+ = \sqrt{(\Upsilon_4 - \Upsilon_2)\,g(f,y)}, \Upsilon_4 > \Upsilon_2;\\ &= \exp[-\tau_-\,(f,y)\,s], \tau_- = \sqrt{(\Upsilon_2 - \Upsilon_4)\,g(f,y)}, \Upsilon_4 < \Upsilon_2;\\ &= |c_1(f,y) + sc_2(f,y)|^2, \Upsilon_4 = \Upsilon_2,\end{aligned}$$

and

$$w_i = -\partial_i(\ln|\rho^*|)/(\ln|\rho^*|)^*,$$
$$n_i = n_{i[0]}(f,y) + n_{i[1]}(f,y)\int \exp[-3\rho]ds,$$

with functions $c_{1,2}(f,y)$ and $n_{i[0,1]}(f,y)$ to be stated by some boundary conditions. We emphasize that the constants k_f and k_y have to be defined from some experimental data.

The solution (10) transforms into the usual RS solution (6) if $k_y = 0, n_{i[0,1]}(f,y) = 0, \Lambda = \Lambda_0 = const$ and $\Upsilon_2 \to \Upsilon_{2[0]} = -\frac{\Lambda_0}{4M^3}$; $\Upsilon_1, \Upsilon_3, \Upsilon_4, \Upsilon_5 \to \Upsilon_{[0]} = \frac{V_{brane}}{4M^3}\delta(f) + \frac{V_{brane'}}{4M^3}\delta(f-\pi r_c)$, which holds only when the boundary and bulk cosmological terms are related by formulas $V_{brane} = -V_{brane'} = 24M^3k_f, \quad \Lambda_0 = -24M^3k_f^2;$ we use values with the index [0] in order to emphasize that they belong to the usual (holonomic) RS solutions. In the anholonomic case with "variation of constants" we shall not impose such relations.

We note that using the metric (10) with anisotropic warp factor (11) it is easy to identify the massless gravitational fluctuations about our classical solutions like in the usual RS cases but performing (in this work) all computations with respect to anholonomic frames. All off–diagonal fluctuations of the anholonomic diagonal metric are massive and excluded from the low-energy effective theory.

We see that the physical mass scales are set by an anisotropic symmetry–breaking scale, $v(y) \equiv e^{-k_y|y|}e^{-k_f r_c \pi}v_0$. This result the conclusion: any mass parameter m_0 on the visible 3-brane in the fundamental higher-dimensional theory with Salam–Strathee –Peracci gauge interactions and/or effective anholonomic frames will correspond to an anisotropic dependence on coordinate y of the physical mass $m(y) \equiv e^{-k_y|y|}e^{-kr_c\pi}m_0$ when measured with the metric $\overline{g}_{\mu\nu}$ that appears in the effective Einstein action, since all operators get rescaled according to their four-dimensional conformal weight. If $e^{kr_c\pi}$ is of order 10^{15}, this mechanism can produces TeV physical mass scales from fundamental mass parameters not far from the Planck scale, 10^{19} GeV. Because this geometric factor is an exponential, we clearly do not require very large hierarchies among the fundamental parameters, v_0, k, M, and $\mu_c \equiv 1/r_c$; in fact, we only require $kr_c \approx 50$. These conclusions were made in Refs. [3] with respect to diagonal (isotropic) metrics. But the physical consequences could radically change if the off–diagonal metrics with effective anholonomic frames and gauge fields are considered. In this case we have additional dependencies on variable y which make the fundamental spacetime geometry to be locally anisotropic, polarized via dependencies both on coordinate y receptivitity k_y. We emphasize that our y coordinate is not that from [3].

The phenomenological implications of these anisotropic scenarios for future collider searches could be very distinctive: the geometry of experiments will play a very important role. In such anisotropic models we also have a roughly weak scale splitting with a relatively small number of excitations which can be kinematically accessible at accelerators.

We also reconsider in an anisotropic fashion the derivation of the 4D effective Planck scale M_{Pl} given in Ref. [3]. The 4D graviton zero mode follows from the solution, Eq. (10), by replacing the Minkowski metric by a effective 4D metric $\overline{g}_{\mu\nu}$ which it is described by an effective action following from substitution into Eq. (7),

$$S_{eff} \supset \int \delta^4x \int_0^{\pi r_c} df\ 2M^3 r_c e^{-2k_f|f|}e^{-2k_y|y|}\sqrt{\overline{g}}\ \overline{R}, \qquad (12)$$

where $\overline{R}$ denotes the four-dimensional Ricci scalar made out of $\overline{g}_{\mu\nu}(x)$, in contrast to the five-dimensional Ricci scalar, R, made out of $G_{MN}(x,f)$. We use the symbol δ^4x in (7) in order to emphasize that our integration is adapted to the anholonomic strucute stated by the differentials (5). We also can explicitly perform the f integral in (12) to obtain a purely 4D action and to derive

$$M_{Pl}^2 = 2M^3 \int_0^{\pi r_c} df e^{-2k_f|f|} = \frac{M^3}{k}e^{-2k_y|y|}[1-e^{-2k_f r_c\pi}]. \qquad (13)$$

We see that there is a well-defined value for M_{Pl}, even in the $r_c \to \infty$ limit, but which may have an anisotropic dependence on one of the 4D coordinates, in the stated parametrizations denoted by y. Nevertheless, we can get a sensible effective anisotropic 4D theory, with the usual Newtonian force law, even in the infinite radius limit, in contrast to the product–space expectation that $M_{Pl}^2 = M^3 r_c \pi$.

In consequence of (13), the gravitational potential behaves anisotropically as

$$V(r) = G_N \frac{m_1 m_2}{r}\left(1 + \frac{e^{-2k_y|y|}}{r^2 k_f^2}\right)$$

i.e. our models produce effective 4D theories of gravity with local anisotropy. The leading term due to the bound state mode is the usual Newtonian potential; the Kaluza Klein anholonomic modes generate an extremely anisotropically

suppressed correction term, for k_f taking the expected value of order the fundamental Planck scale and r of the size tested with gravity.

Let us conclude the paper: It is known that we can consistently exist with an infinite 5th dimension, without violating known tests of gravity [3]. The scenarios consist of two or a single 3-brane, (a piece of) AdS_5 in the bulk, and an appropriately tuned tension on the brane. But if we consider off–diagonal 5D metrics like in Ref. [6], which was used for including of $U(1), SU(2)$ and $SU(3)$ gauge fields, or, in a different but similar fashion, for construction of generic anisotropic, partially anholonomic, solutions (like static black holes with ellipsoidal horizons, static black tori and anisotropic wormholes) in Einstein and extra dimension gravity, [7] the RS theories become substantially locally anisotropic. One obtains variations of constants on the 5th coordinate and possible anisotropic oscillations in time (in the first our model), or on space coordinate (in the second our model). Here it should be emphasized that the anisotropic oscillations (in time or in a space coordinate) are defined by the constant component of the cosmological constant (which in our model can generally run on the 5th coordinate). This sure is related to the the cosmological constant problem which in this work is taken as a given one, with an approximation of linear dependence on the 5th coordinate, and not solved. In the other hand a new, anisotropic, solution to the hierarchy problem is supposed to be subjected to experimental verification.

Finally, we note that many interesting questions remain to be investigated. Having constructed another, anisotropic, valid alternative to conventional 4D gravity, it is important to analyze the astrophysical and cosmological implications. These anisotropic scenarios might even provide a new perspective for solving unsolved issues in string/M-theory, quantum gravity and cosmology.

The author thanks D. Singleton, E. Gaburov and D. Gontsa for collaboration and discussing of results. The author is grateful to P. Stavrinos for hospitality and support. The work is partially supported by "The 2000–2001 California State University Legislative Award".

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